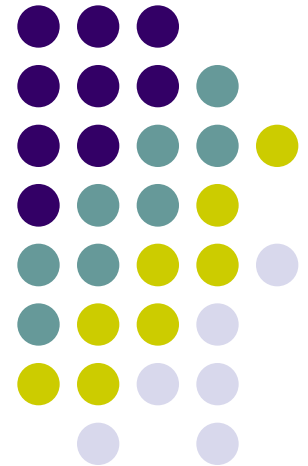
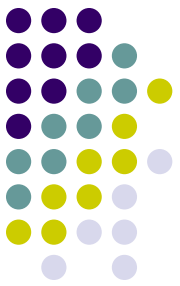


SIGNAL DIGITIZATION

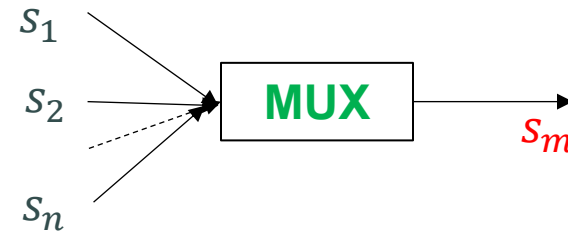
EEEN 464 - DIGITAL
COMMUNICATION
Tuesday, June 3, 2025



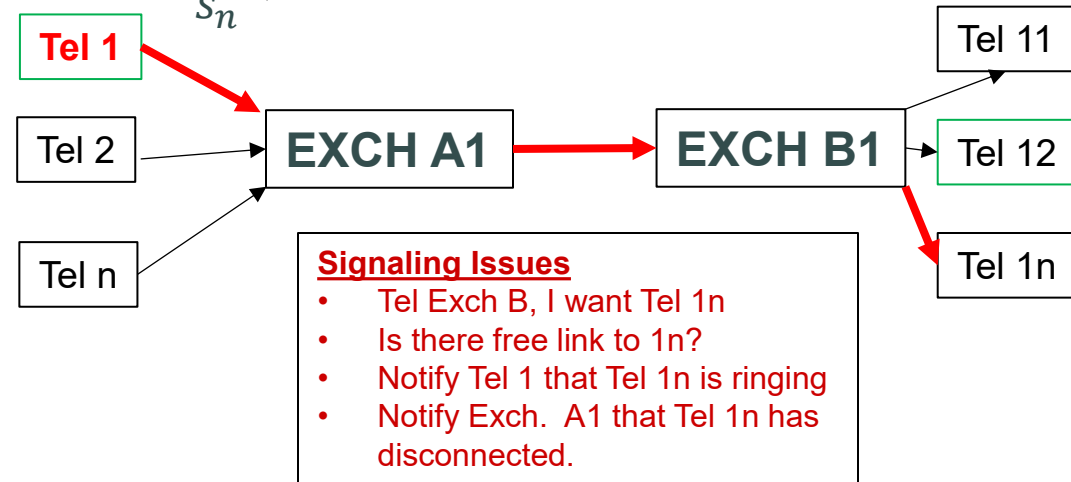
ADVANTAGES OF DIGITAL COMMUNICATION NETWORKS /1



1. Ease of multiplexing



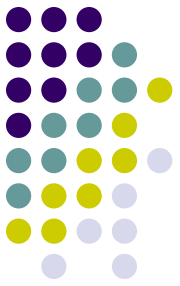
2. Ease of Signaling



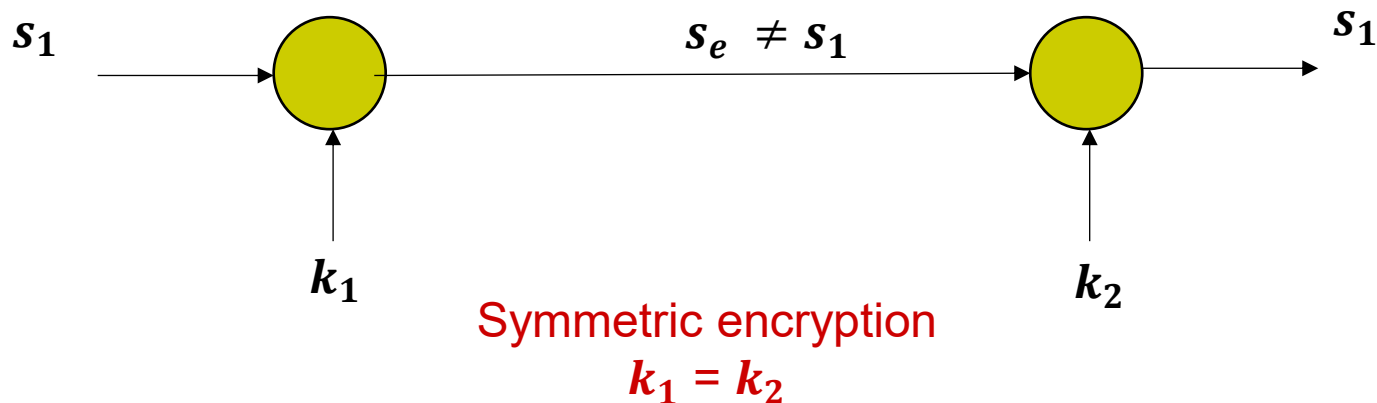
3. Uses modern technology

4. Integration of transmission and switching

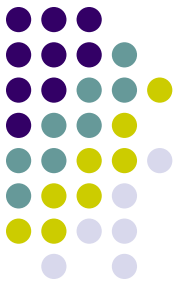
5. Signal Regeneration



6. Advanced Performance Monitoring.
7. Ability to integrate other services.
8. Ability to operate at low Signal to Noise Ratio.
9. Ease of encryption.

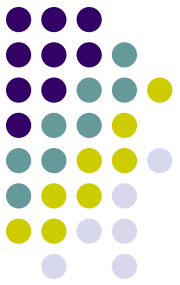


COST OF MULTIPLEXING



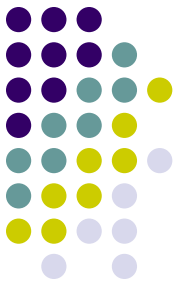
- The Cost of digital multiplex systems, e.g. TDM is much lower than that of analogue multiplex systems, e.g. FDM.

SIGNALLING



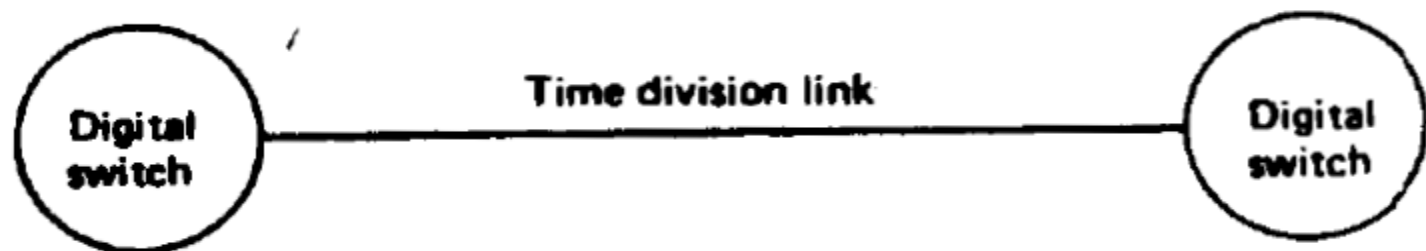
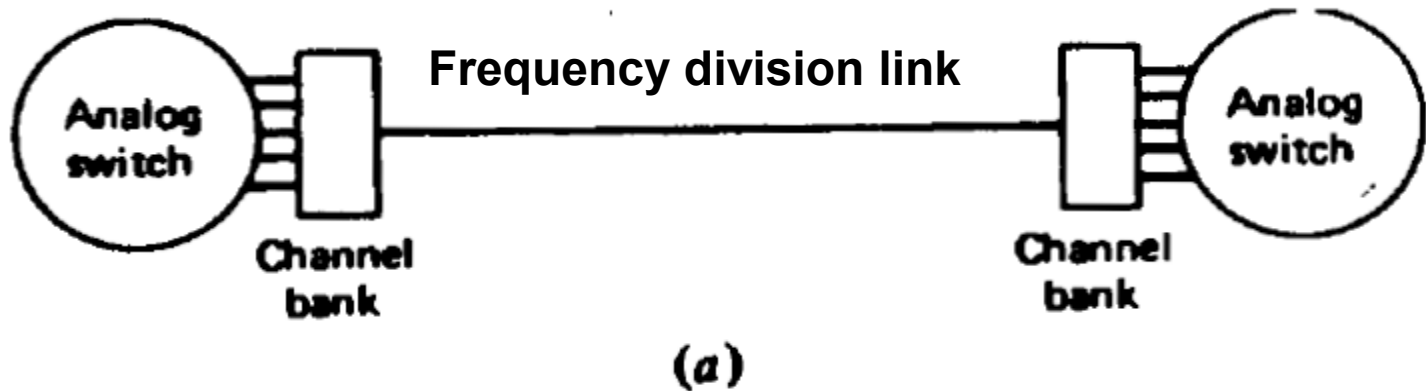
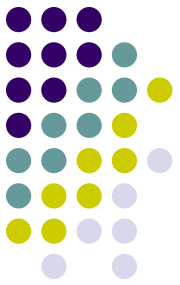
1. Digital systems allow **control information to be inserted into and extracted** from a message stream independent of the mode of transmission.
2. **Signaling equipment can therefore be designed separate from transmission systems** allowing control functions and formats to be designed and modified independently.

USE OF MODERN TECHNOLOGY

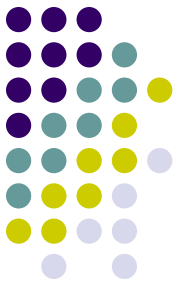


1. Multiplexer and switching matrix for digital systems are implemented with the **same basic circuits used in computers.**
2. **Special LSI Circuits** have been developed specifically for telecommunication functions e.g Voice Codecs, Multiplexing, DSPs, etc.
3. **Low-cost of digital circuitry** allow for implementations that would be very expensive if developed on analogue platforms, e.g large non-blocking exchanges.
4. Digital technology provides **easier and cheaper interfaces to fibre-optic cable systems.**

INTEGRATION OF TRANSMISSION AND SWITCHING`

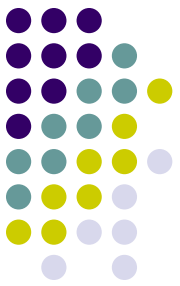


BENEFITS OF FULLY INTEGRATED DIGITAL NETWORKS

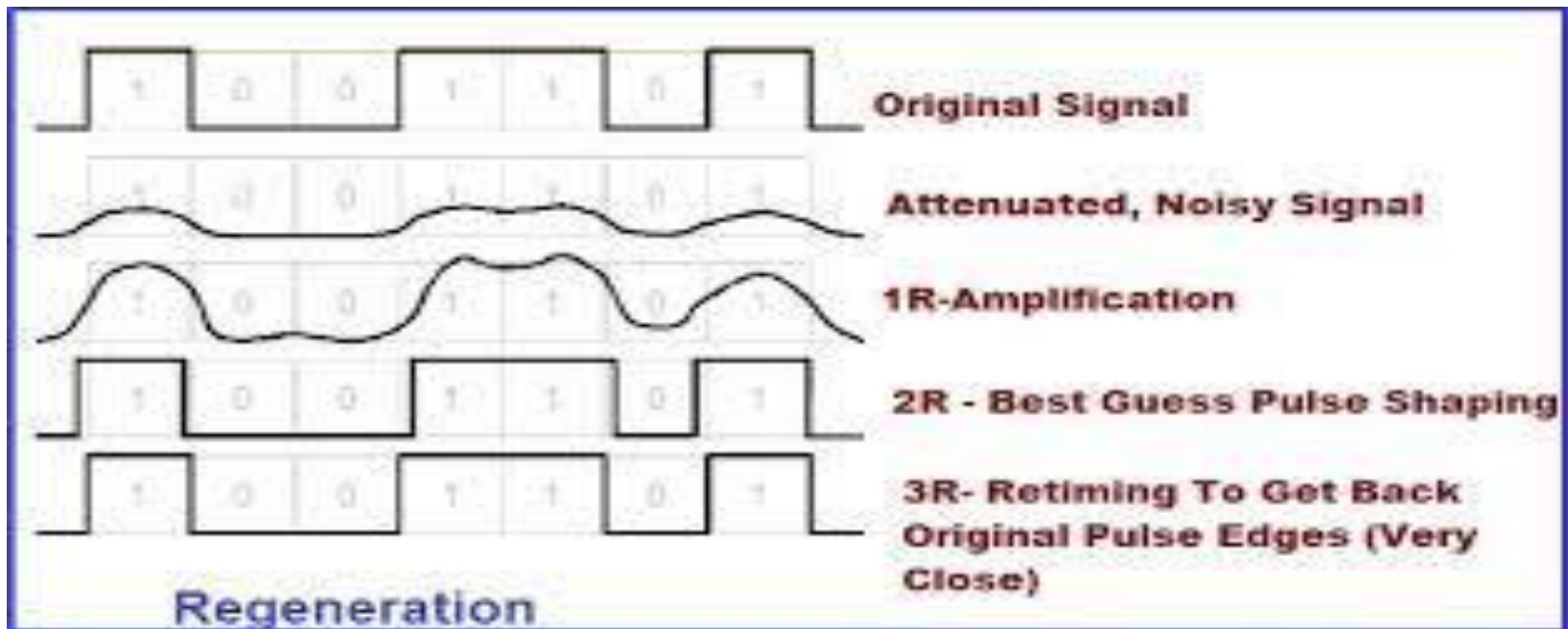
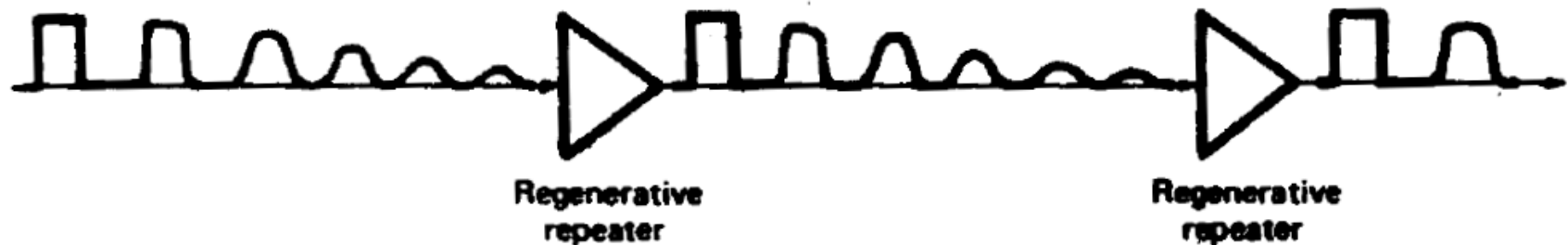


1. Long-distance and local voice quality are identical in terms of noise, signal level, and distortion.
2. Since digital baseband circuits are inherently four-wire, network-generated echoes are eliminated, and true full-duplex, four-wire digital circuits are available.
3. Cable entrance requirements and distribution of wire pairs is greatly reduced because all trunks are implemented as sub-channels in a TDM signal.

SIGNAL REGENERATION



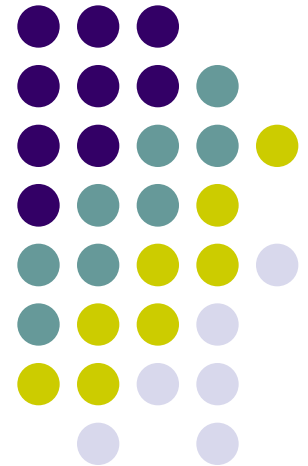
Digital Signals can be **regenerated** at suitable intervals **unlike analogue signals**.



SPEECH DIGITIZATION

ECE416

Thursday, October 5, 2023

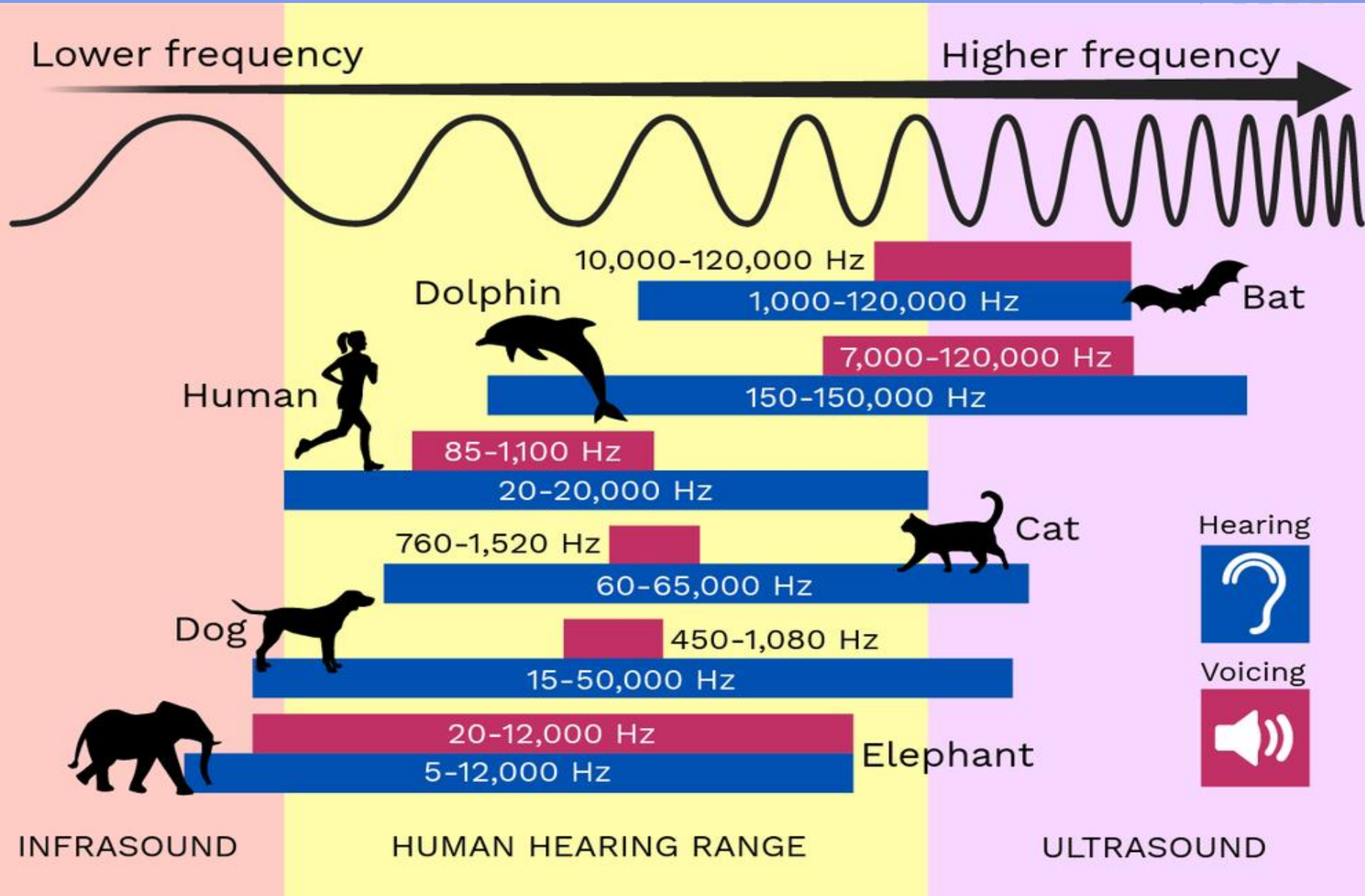


HUMAN HEARING & VOICING FREQUENCY RANGE

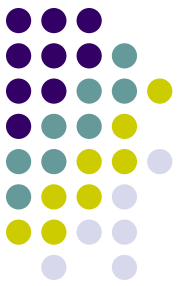


1. Human ear can detect sounds with frequencies between 20 Hz and 20,000 Hz.
2. Sound with frequencies below the human hearing range is referred to as infrasound.
3. Sound with frequencies above is called ultrasound.
4. The range of frequencies that humans can produce is called the voicing range.
5. Voicing range varies from 85 Hz to 1,100 Hz.

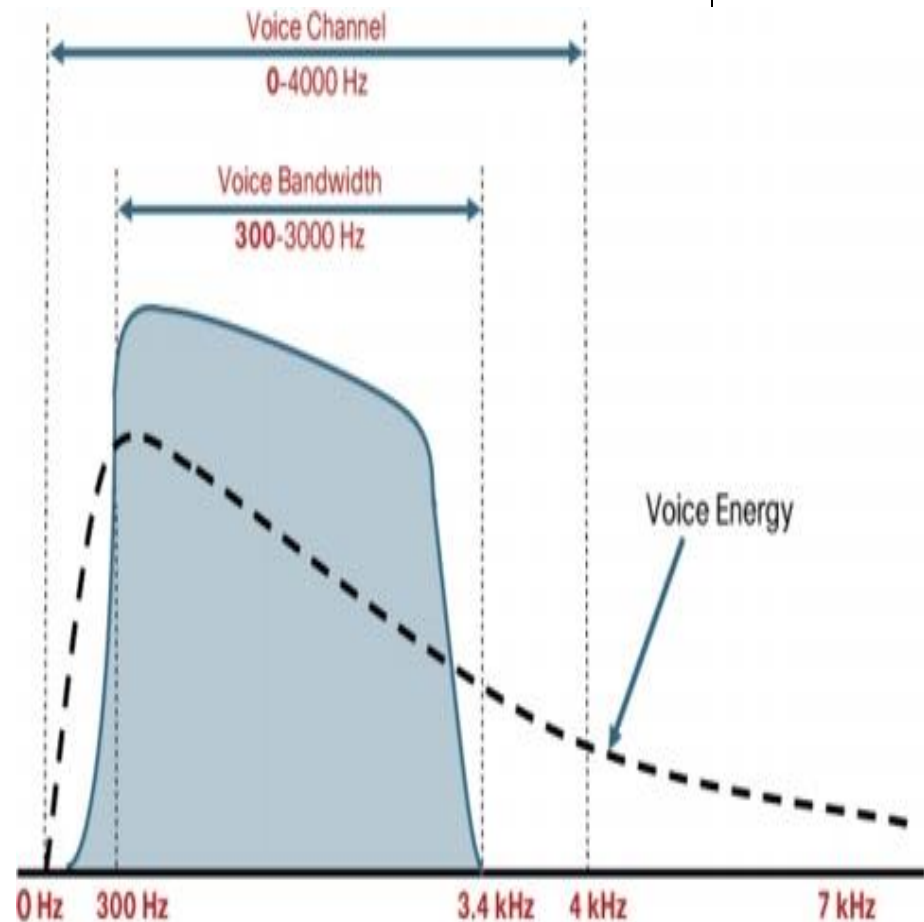
HEARING & VOICING AMONGST SPECIES



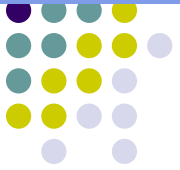
TELEPHONE FREQUENCY RANGE



- The range of frequencies that the human ear can perceive **20Hz – 20KHz** (Natural voice frequency range)
- Acceptable level of intelligibility is obtained by transmitting voice in range **0.3 -3.4 KHz**
- Most of the voice energy is in this band.



SAMPLING THEORY



Sampling is the process of selecting portions of a continuous-time signal to form a discrete-time signal based on two criteria:

- (a) There should be sufficient number of samples so that the original signal is adequately represented.
- (b) It should be possible to reconstruct the original signal from its samples.

SAMPLING THEORY



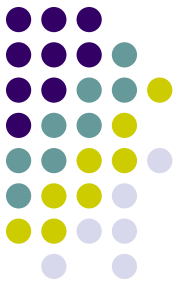
Sampling theorem can be stated by any of the following:

1. A signal that is band-limited of finite energy at a frequency f_m can be **completely described by samples taken at a uniform time intervals of no less than $\frac{1}{2f_m}$ apart.**
2. A band-limited signal of finite energy with no frequency above f_m may be **adequately recovered from samples taken at the rate $2f_m$ samples per second.**

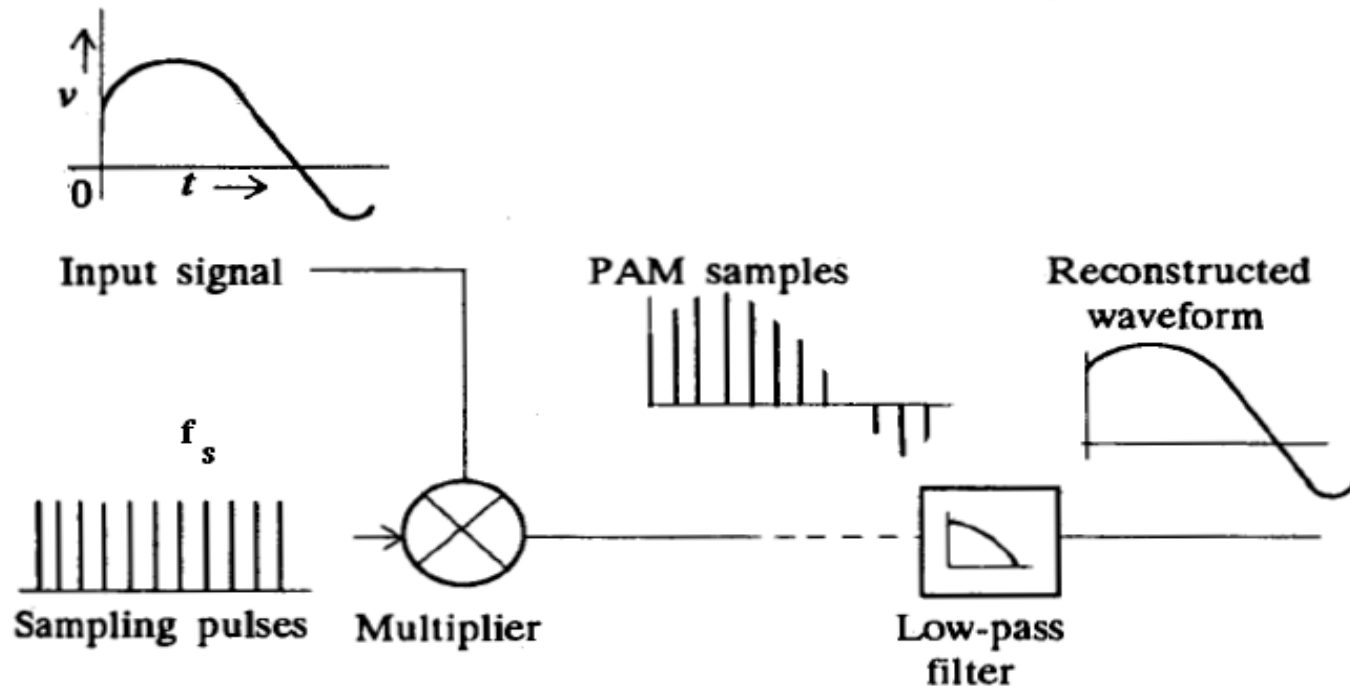
MATCHING THEORY TO SAMPLING REQUIREMENTS

REQUIREMENT	SAMPLING THEORY
1. There should be sufficient number of samples so that the original signal is adequately represented	A signal that is band-limited of finite energy at a frequency f_m can be completely described by samples taken at a uniform time intervals of no less than $\frac{1}{2f_m}$ apart.
2. It should be possible to reconstruct the original signal from its samples.	A band-limited signal of finite energy with no frequency above f_m may be adequately recovered from samples taken at the rate $2f_m$ samples per second.

PULSE-AMPLITUDE MODULATION



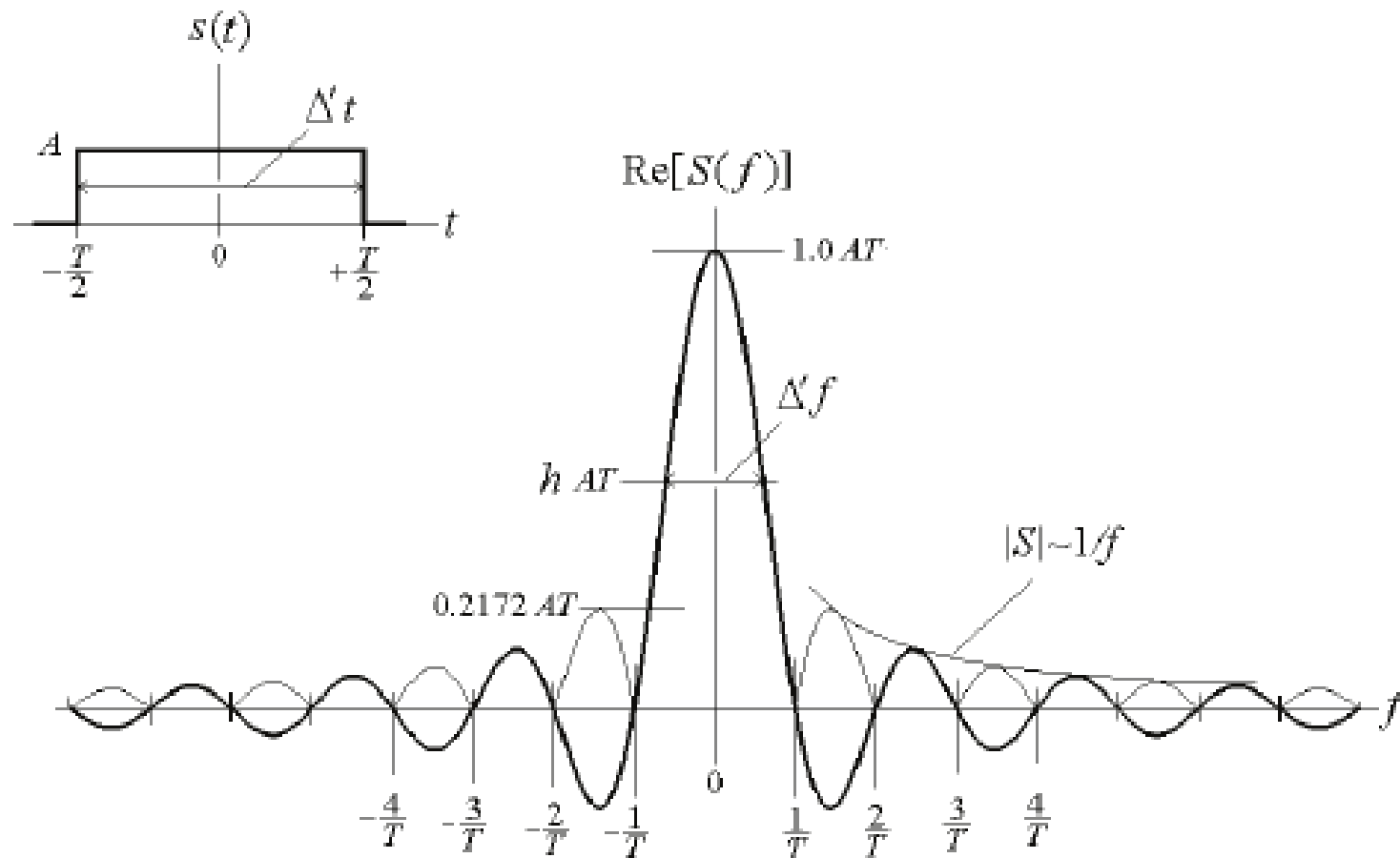
Pulse-amplitude modulation (PAM), is a form of signal modulation where the message information is encoded in the amplitude of a series of signal samples.



Nyquist Criterion/Theorem

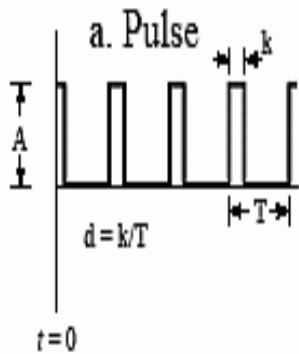
- $f_s > 2f_{\max}$ where $f_{\max}$ is the highest frequency in the analog input signal

RECAP: FOURIER TRANSFORM OF A PULSE

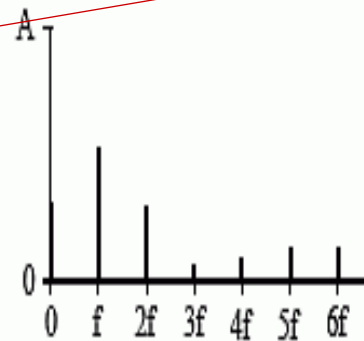


RECAP: FOURIER TRANSFORM OF A PULSE TRAIN

Time Domain



Frequency Domain



$$a_0 = A d$$

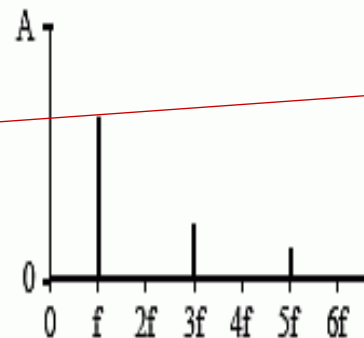
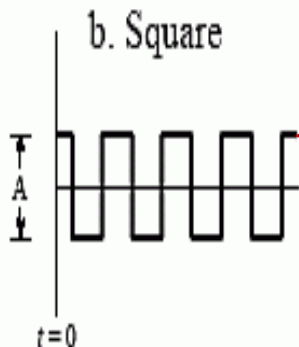
$$a_n = \frac{2A}{n\pi} \sin(n\pi d)$$

$$b_n = 0$$

($d = 0.27$ in this example)

Duty cycle of 0.27: The spectral content at closest to $3f$ is quite small.

- At a duty-cycle of exactly one-third, the spectral content at $3f$ would be zero.



$$a_0 = 0$$

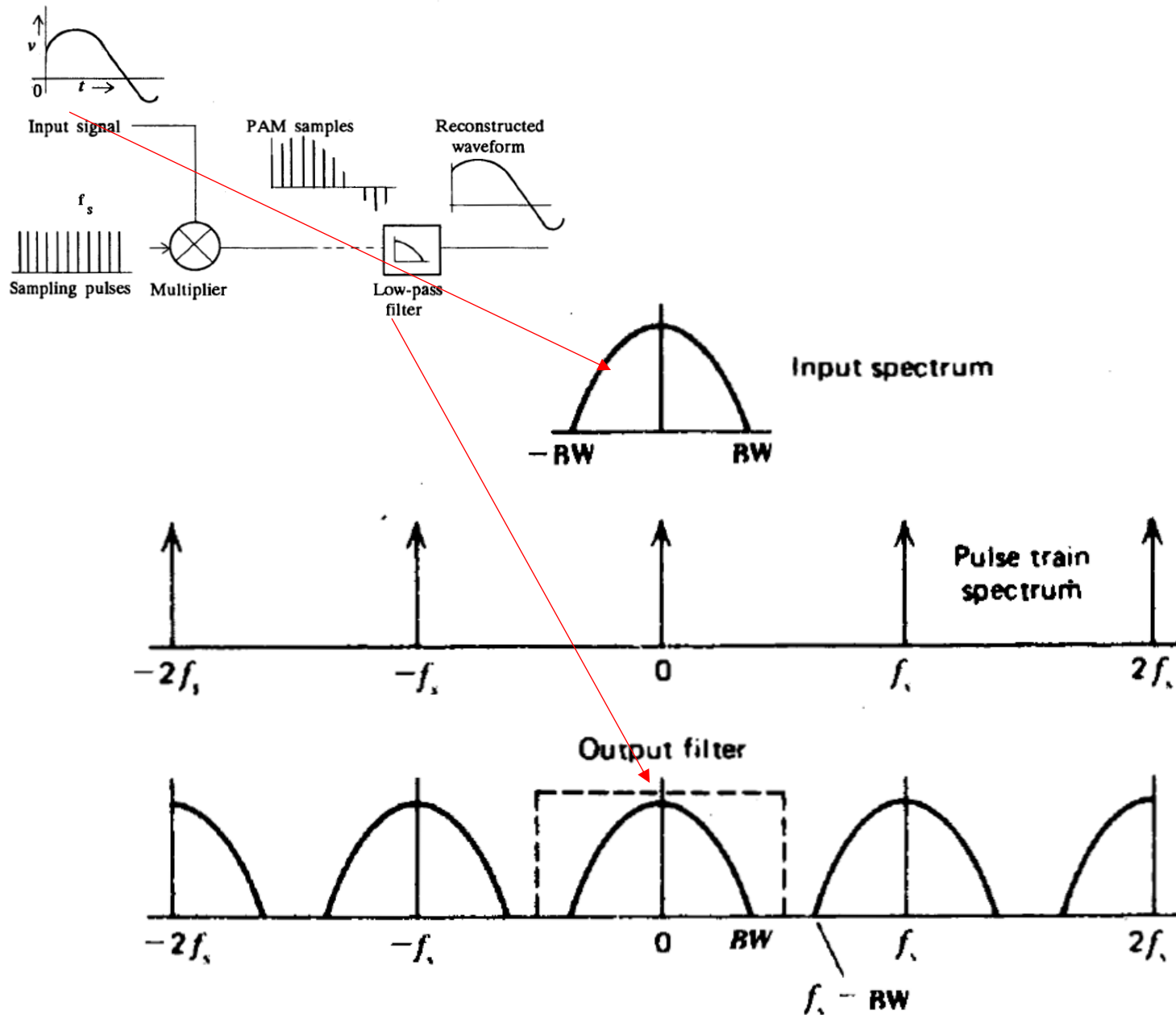
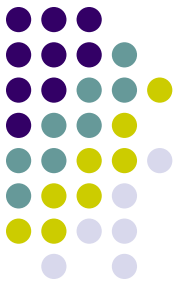
$$a_n = \frac{2A}{n\pi} \sin\left(\frac{n\pi}{2}\right)$$

$$b_n = 0$$

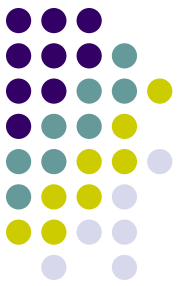
(all even harmonics are zero)

Duty cycle is 0.5: The spectral content at $2f$ (and $4f$ and $6f$ etc..) is always zero.

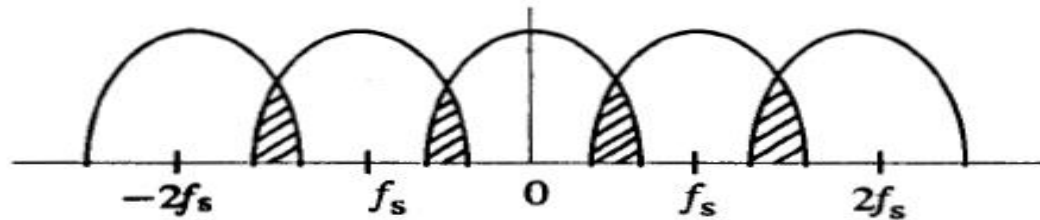
PULSE AMPLITUDE MODULATION (PAM) SPECTRUM



ALIASING/FOLD-OVER DISTORTION

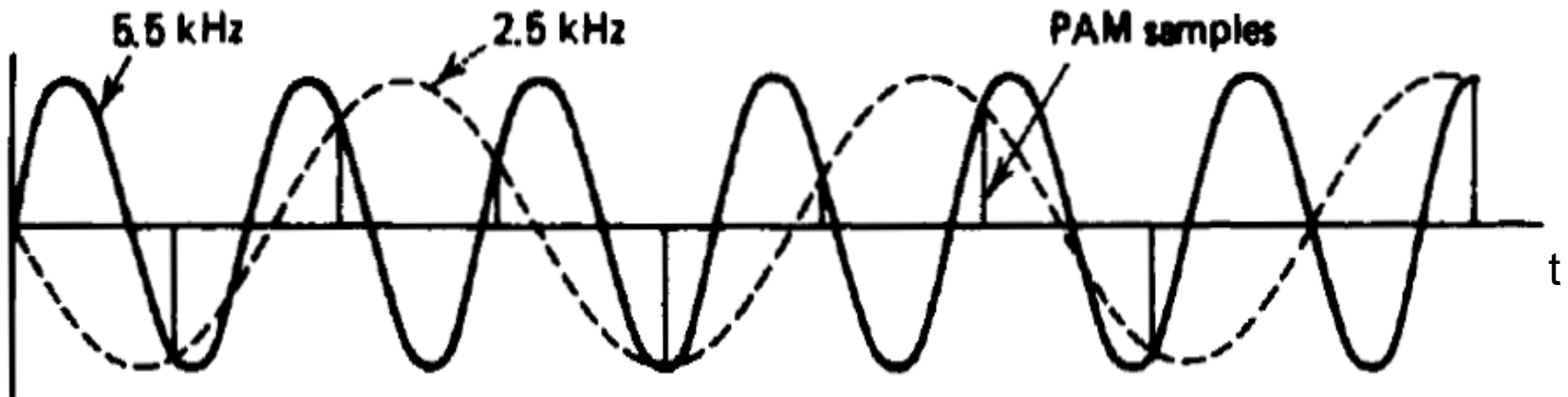
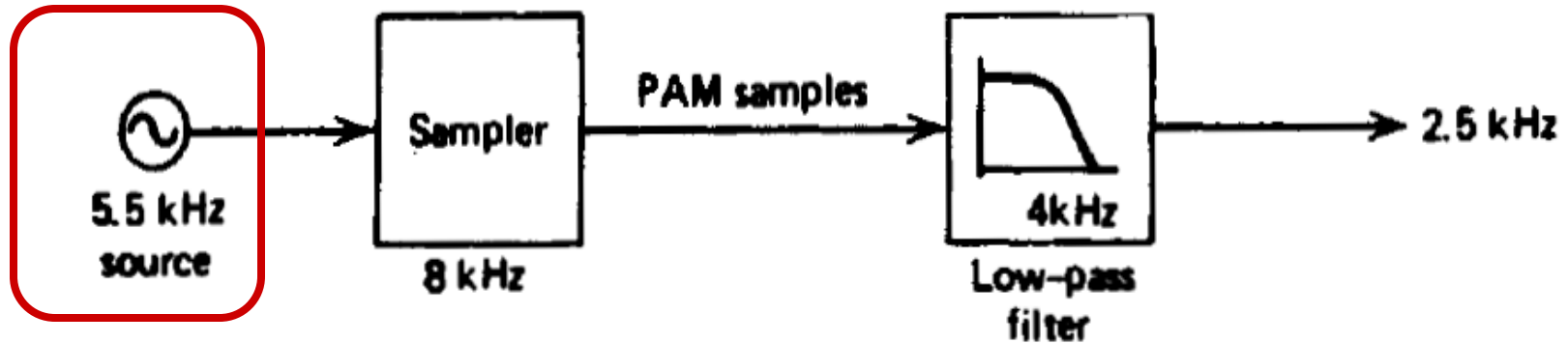


1. **Aliasing/fold-over** occurs when the sampling frequency f_s is less than the Nyquist frequency, $2f_{\max}$ resulting in an overlap of the spectrum.



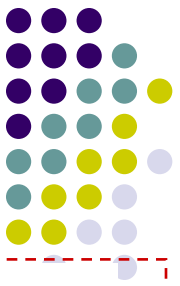
2. Aliasing/fold-over distortion is avoided in traditional telephony by:
 1. Band-limiting the signal to the range 0.3-3.4 KHz.
 2. over-sampling at 8KHz, the sampled signal is sufficiently attenuated at the overlap frequency of 4KHz

ILLUSTRATION OF ALIASING DISTORTION

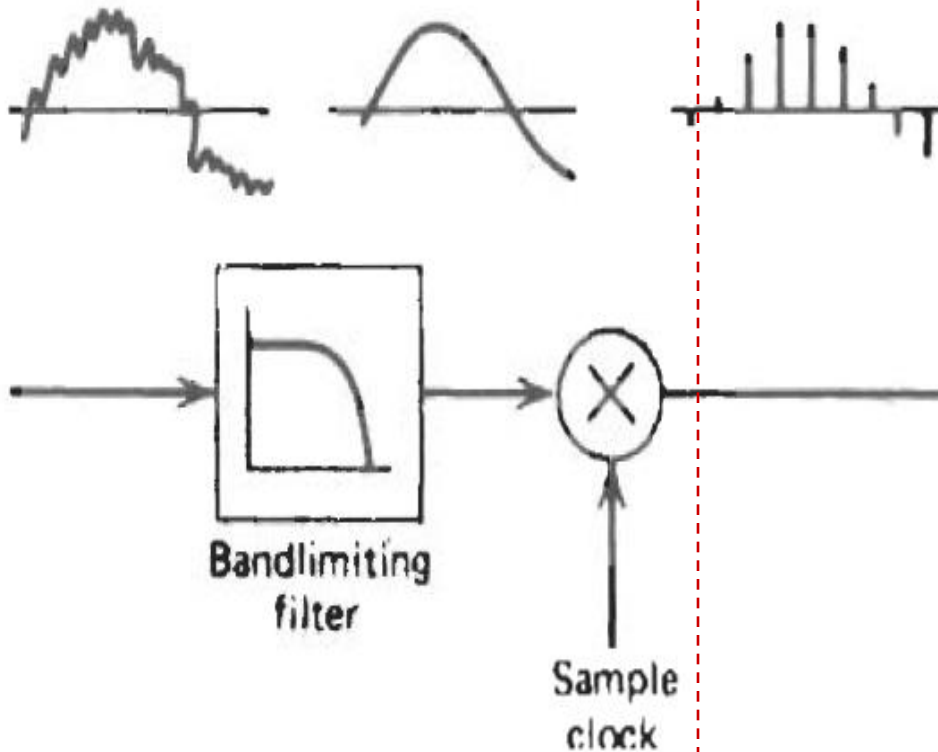


Signal of 5.5 KHz sampled at 8 KHz may appear to be a 2.5 KHz signal

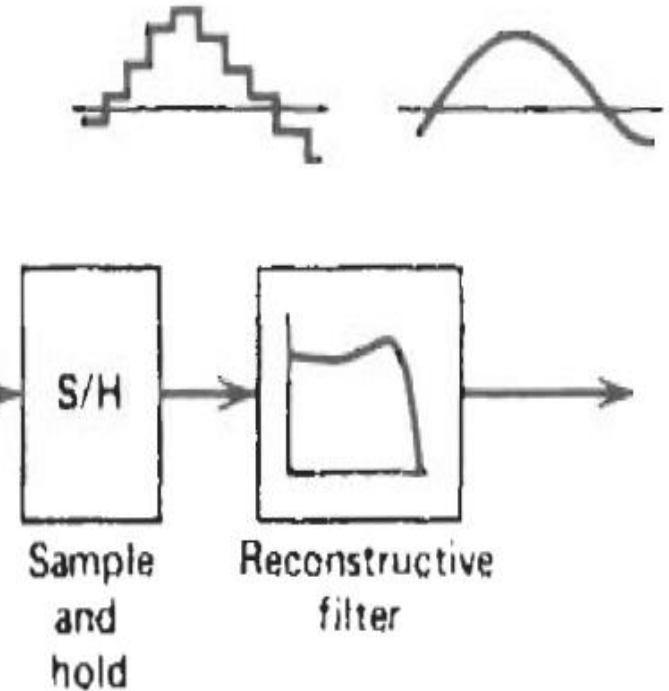
END-TO-END PAM SYSTEM



Transmitter

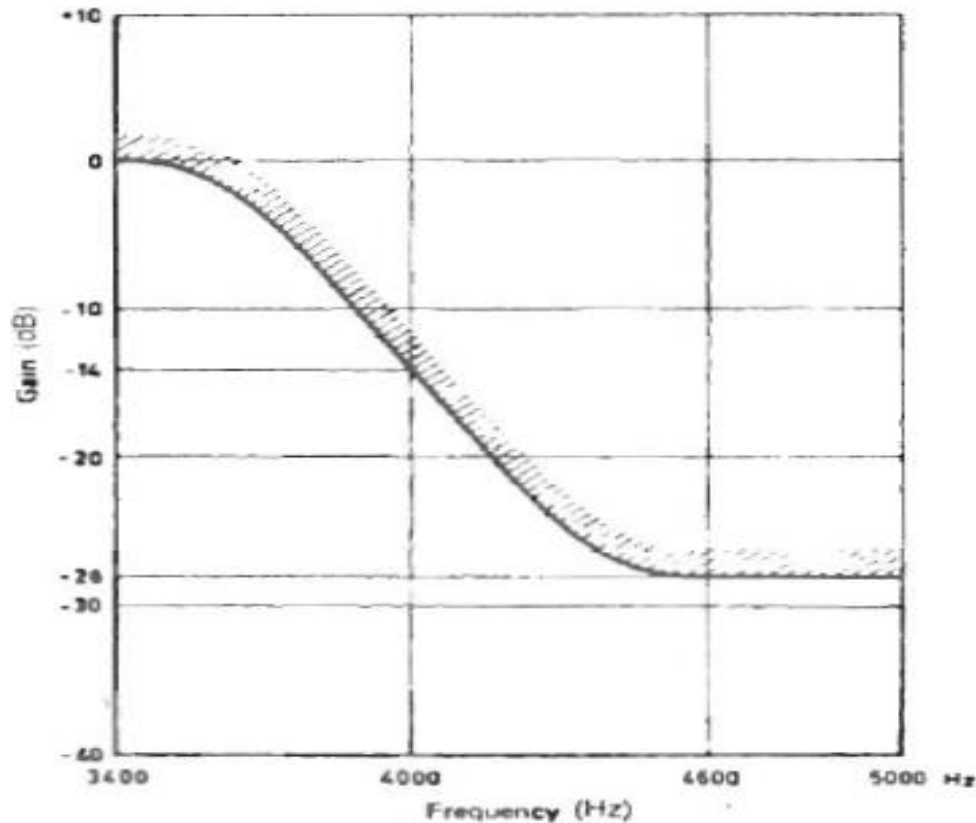
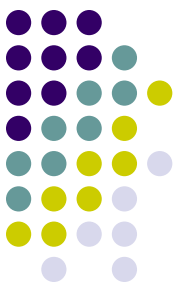


Receiver



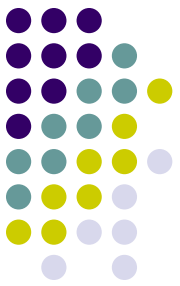
The response of the reconstructive filter is usually modified to account for the spectrum of the wider staircase samples.

BAND-LIMITING FILTER DESIGNED TO MEET INTERNATIONAL TELECOMMUNICATION UNION (ITU) RECOMMENDATION FOR PCM VOICE CODERS

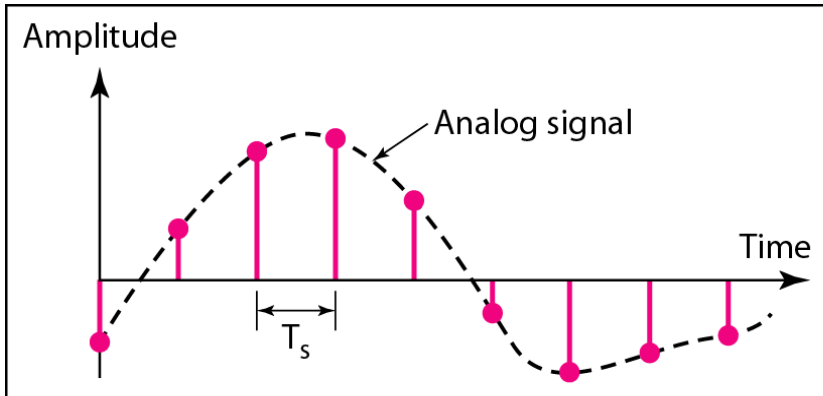


- ITU Standard for telephone speech sampling requires 14 dB attenuation is provided at 4 KHz.

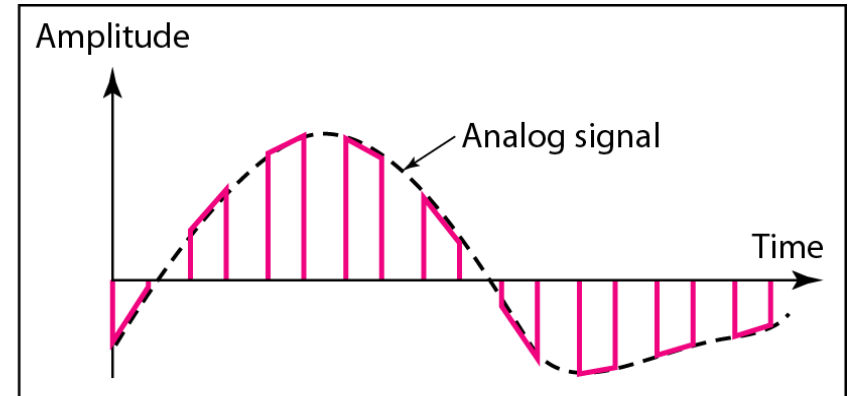
SAMPLING TECHNIQUES



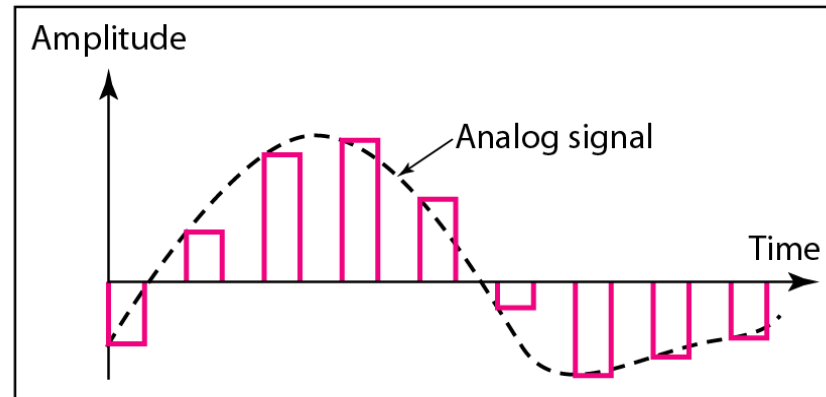
There are three sampling techniques, i.e Ideal, Natural and Flat-top



a. Ideal sampling

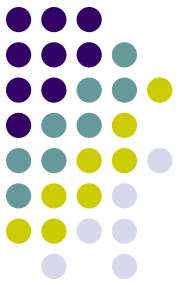


b. Natural sampling

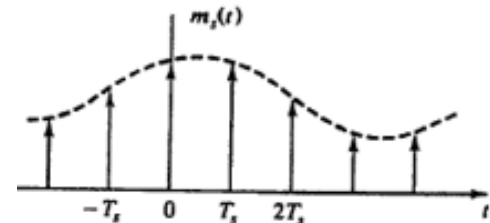
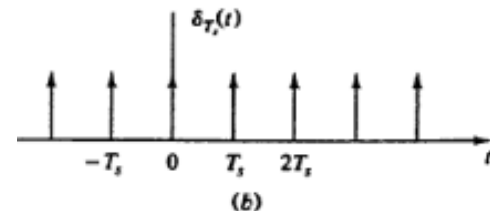
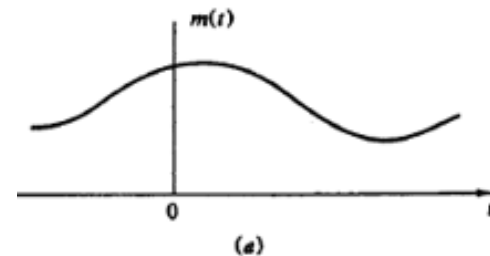
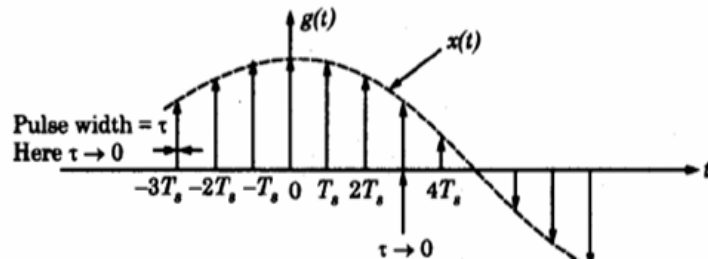
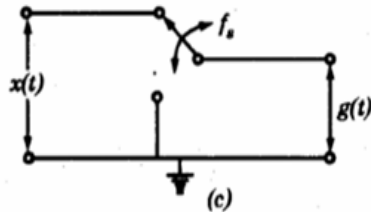
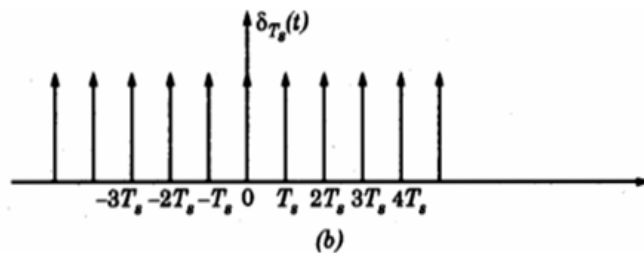


c. Flat-top sampling

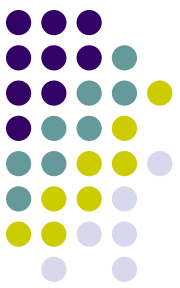
IDEAL SAMPLING



- **Ideal Sampling (Instantaneous sampling or Impulse Sampling)** uses a train of impulses and the principle used is known as multiplication principle.



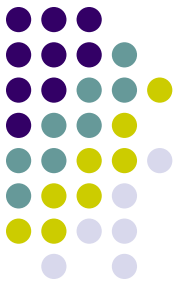
NYQUIST RATE



- **Nyquist rate** is the sampling rate which is exactly equal to $2f_m$ and is the minimum sampling rate required to represent and reconstruct the signal.
- **Nyquist interval** is the interval between and is given by:

$$T_s = \frac{1}{2f_m}$$

EXAMPLE 1



Calculate the Nyquist rate for an analogue signal given by:

$$x(t) = 3 \cos(50\pi t) + 10 \sin(300\pi t) - \cos(100\pi t)$$

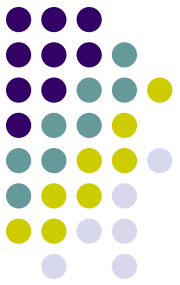
Solution:

$$x(t) = 3 \cos(2\pi f_1 t) + 10 \sin(2\pi f_2 t) - \cos(2\pi f_3 t)$$

$$x(t) = 3 \cos(2\pi 25t) + 10 \sin(2\pi 150t) - \cos(2\pi 50t)$$

Maximum frequency is therefore 150Hz and the maximum Nyquist rate is $f_s = 2 \times 150 = 300$ Samples/Sec

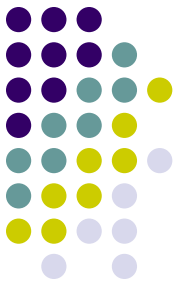
EXAMPLE 2



Find the Nyquist frequency/rate and the Nyquist interval for the following signal:

$$x(t) = \frac{1}{2\pi} \cos(4000\pi t) \cos(1000\pi t)$$

SOLUTION



$$2 \cos(A) \cos(B) = \cos(A + B) + \cos(A - B)$$

$$x(t) = \frac{1}{2\pi} \cos(4000\pi t) \cos(1000\pi t)$$

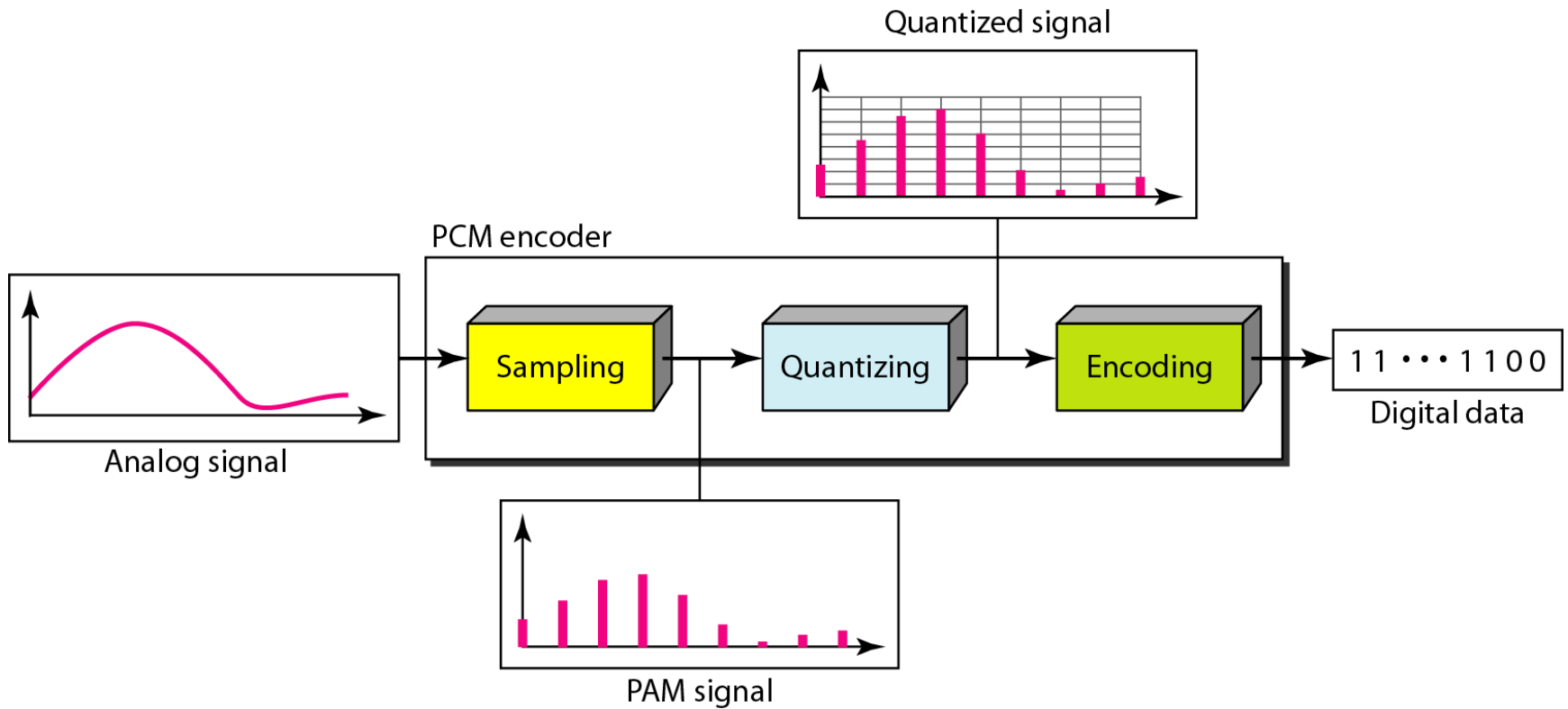
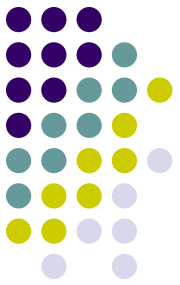
$$x(t) = \frac{1}{4\pi} (\cos(4000\pi t + 1000\pi t) + \cos(4000\pi t - 1000\pi t))$$

or

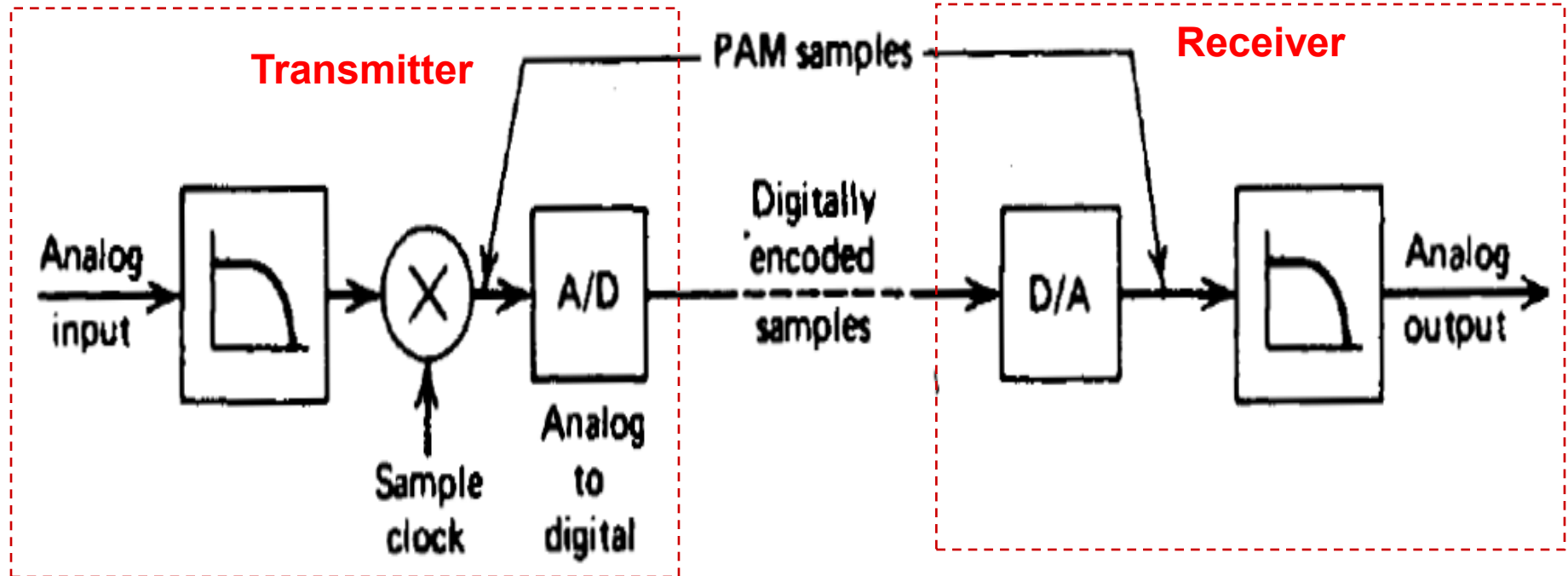
$$x(t) = \frac{1}{4\pi} (\cos(5000\pi t) + \cos(3000\pi t))$$

The maximum frequency is therefore 2,500Hz which implies that the Nyquist rate is 5,000 samples/sec and Nyquist interval is $1/5000 = 0.2\text{msec}$

PULSE CODE MODULATION (1)

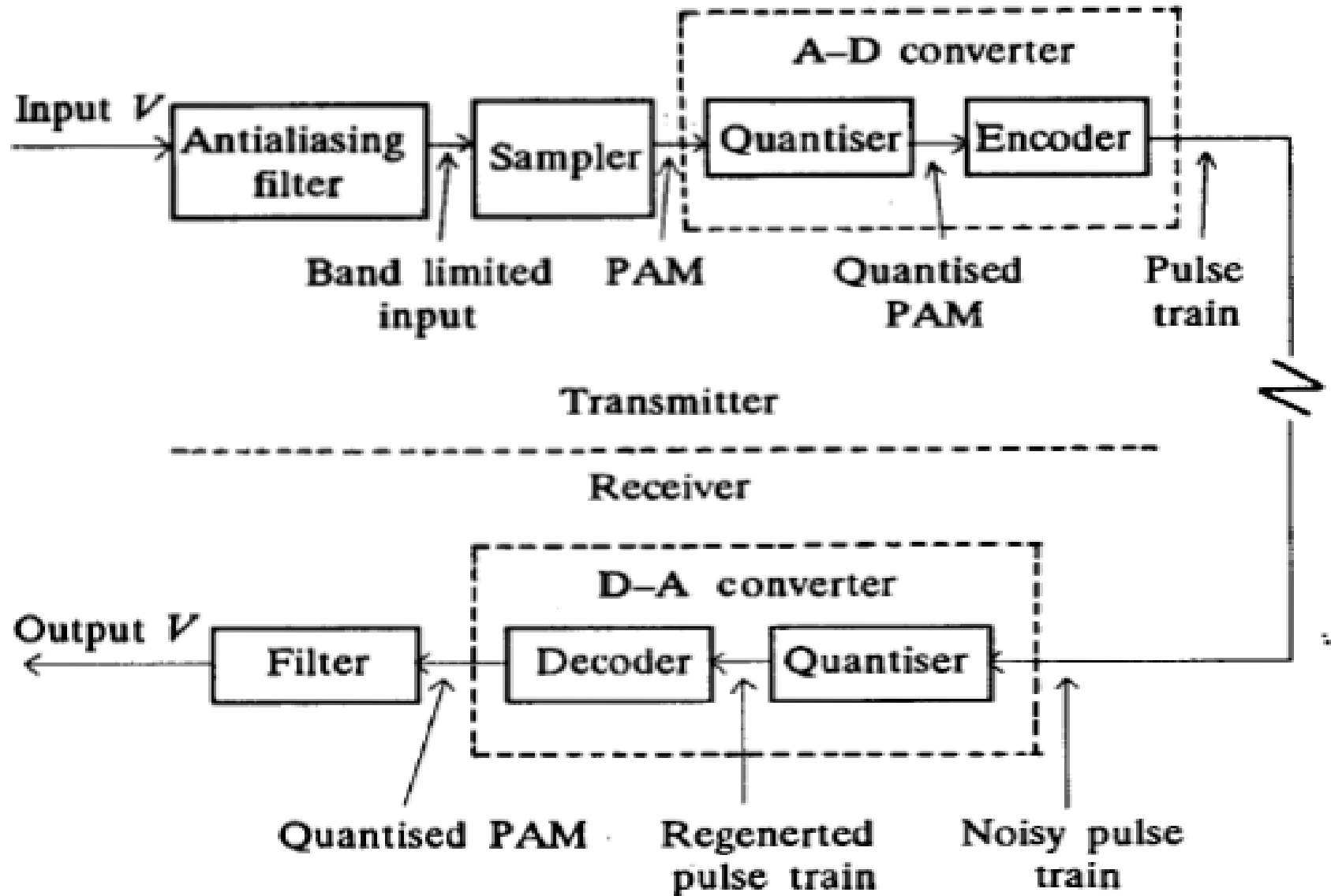


PULSE CODE MODULATION (2)



- **A/D Converters and D/A Converters** are inserted in a PAM system discussed above
- Each sample is represented in binary by **one discrete value**.

PCM SYSTEM FOR SPEECH COMMUNICATION



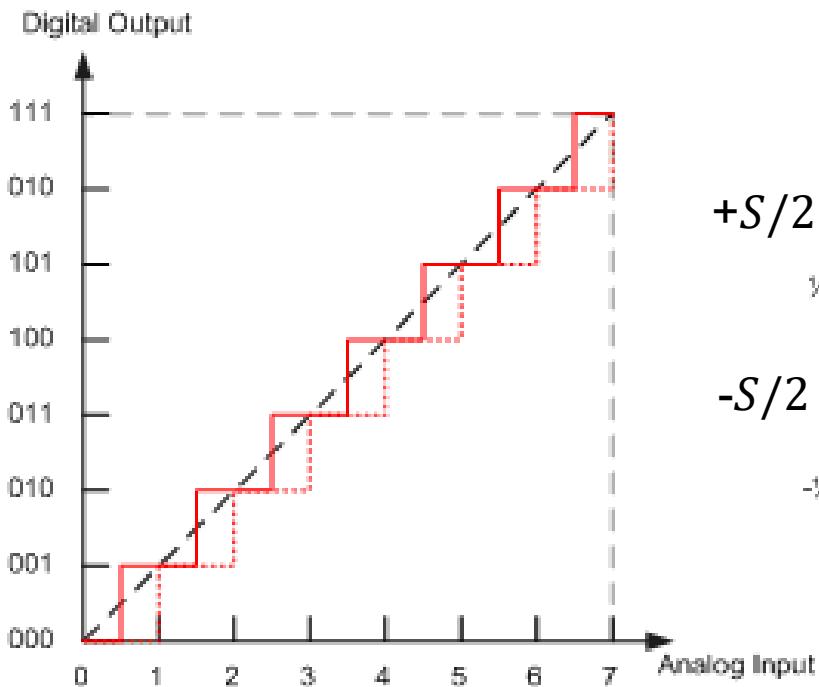
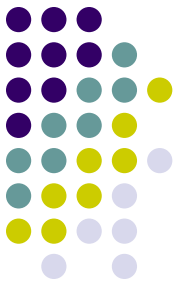
WORKED EXAMPLE

- A PCM telephone system has a band-limiting filter of 4 KHz. If each sample is sampled, quantized at 256 levels, calculate the bit-rate of the PCM system.

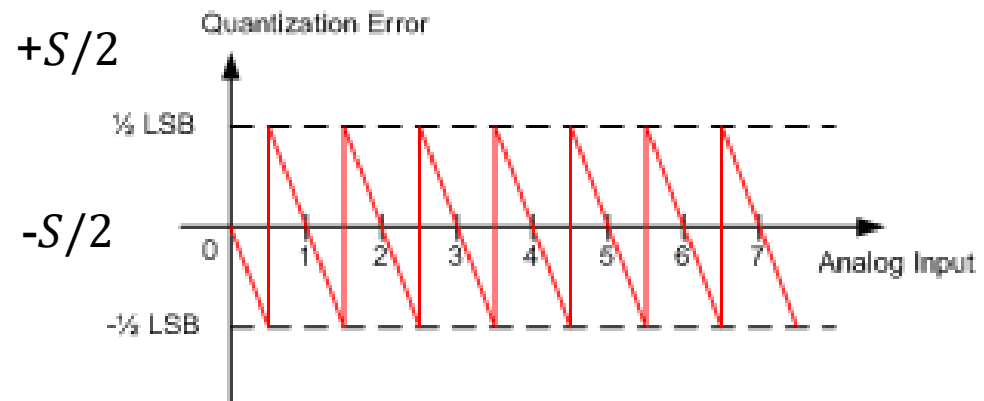
ANS:

1. According to the Nyquist criterion, we must sample at twice the maximum baseband frequency = $2 \times 4 \text{ KHz}$ or 8,000 samples/sec.
2. To represent 256 quantized level, the system requires n bits to represent each sample, where $2^n = 256$ or $n = 8$.
3. The PCM bit rate is therefore $8 \times 8,000 = 64 \text{ Kbits/s}$

QUANTIZATION ERROR AS A FUNCTION OF AMPLITUDE



(a)

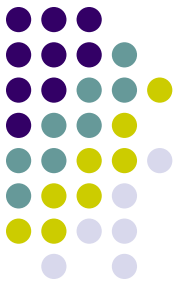


(b)

Quantization Error is $\epsilon = v - v_q$

$$-\frac{S}{2} \leq \epsilon \leq \frac{S}{2}$$

LINEAR QUANTISATION

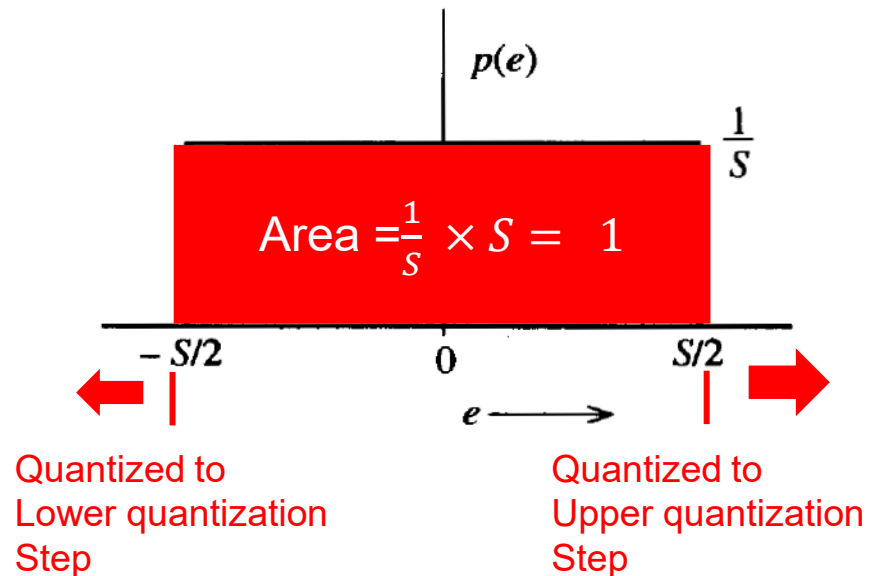


- Assume that the probability distribution of error is constant **within the range $\pm S/2$** .
- Average quantization noise is given by:

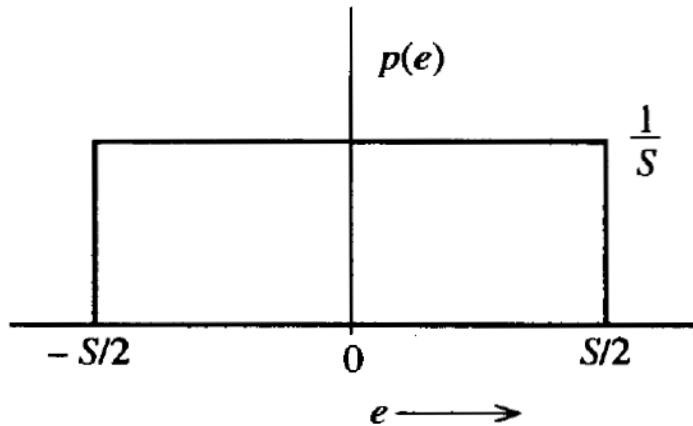
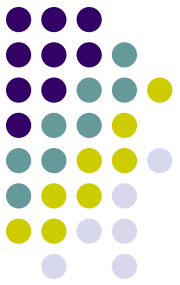
$$\sigma^2 = \int_{-\infty}^{\infty} (e - \mu)^2 p(e) de$$

Where μ = mean

- The range of the quantization error, i.e. $\pm \frac{S}{2}$ forms the limits of integration



AVERAGE QUANTIZATION NOISE POWER FOR LINEAR QUANTISATION

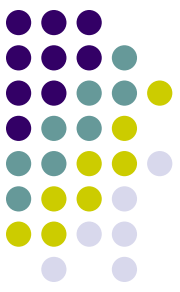


$$\sigma^2 = \int_{-\infty}^{\infty} (e - \mu)^2 p(e) de$$

$$\sigma^2 = \int_{-S/2}^{S/2} (e - 0)^2 \frac{1}{S} de$$

$$= \frac{1}{S} \int_{-S/2}^{S/2} e^2 de$$

$$= \frac{1}{S} \left(\frac{e^3}{3} \right)_{-S/2}^{S/2} = \frac{S^2}{12}$$



SIGNAL TO QUANTISATION NOISE RATIO (SQR)

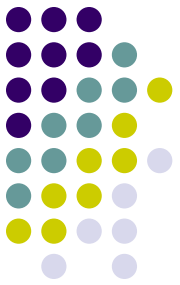
Signal to Quantization Ratio (SQR) is a measure of the performance of a PCM system.

$$\text{SQR} = \frac{E\{x^2(t)\}}{E\{[y(t) - x(t)]^2\}}$$

- Where
 - $E\{..\}$ is the expectation or average
 - $x(t)$ is the analog input signal
 - $y(t)$ is decoded output signal

Assumptions

1. Error $y(t) - x(t)$ is limited in amplitude to $S/2$ where S is the height of the quantisation interval.
2. Sample value is equally likely to fall anywhere in the quantisation interval.
3. Signal amplitude is confined to the maximum range of the coder.



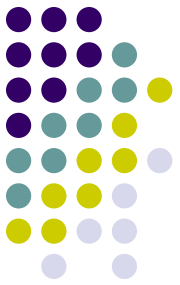
SIGNAL TO QUANTISATION NOISE RATIO (SQR)

4. If all quantization intervals have equal length, i.e uniform quantization, **the quantisation noise is independent of the sample values** and SQR can be determined as:

$$\begin{aligned} \text{SQR (db)} &= 10 \log_{10} \left(\frac{v^2}{q^2/12} \right) \\ &= 10.8 + 20 \log_{10} \left(\frac{v}{q} \right) \end{aligned}$$

Where **v** is the rms value of the input and **q** is the quantization noise.

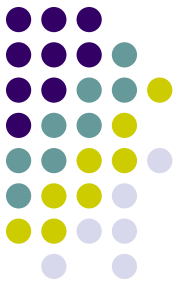
SQR FOR SINEWAVE



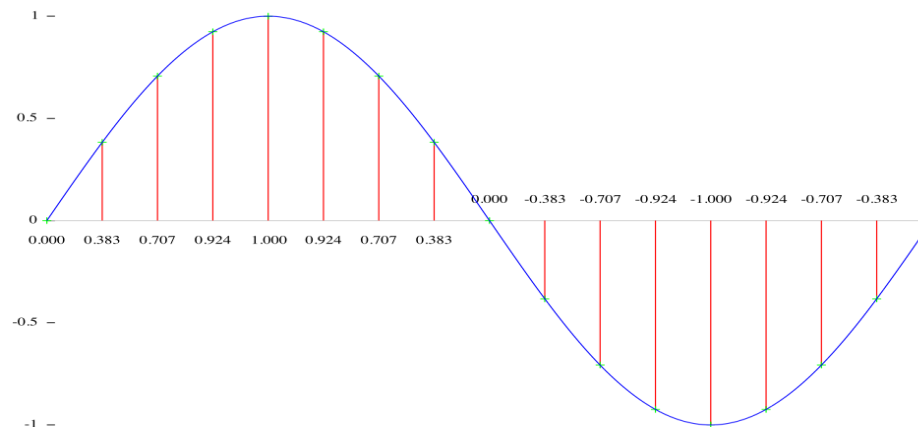
- For a sinewave with Peak Amplitude A, SQR is given by:

$$\begin{aligned}\text{SQR (dB)} &= 10 \log_{10} \left(\frac{A^2/2}{q^2/12} \right) \\ &= 7.78 + 20 \log_{10} \left(\frac{A}{q} \right)\end{aligned}$$

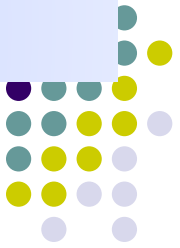
EXAMPLE



- A sine wave with a 1-V maximum amplitude is to be digitized with a minimum SQR of 30dB.
- How many uniformly spaced quantization intervals are needed and how many bits are needed to encode each sample?



SOLUTION

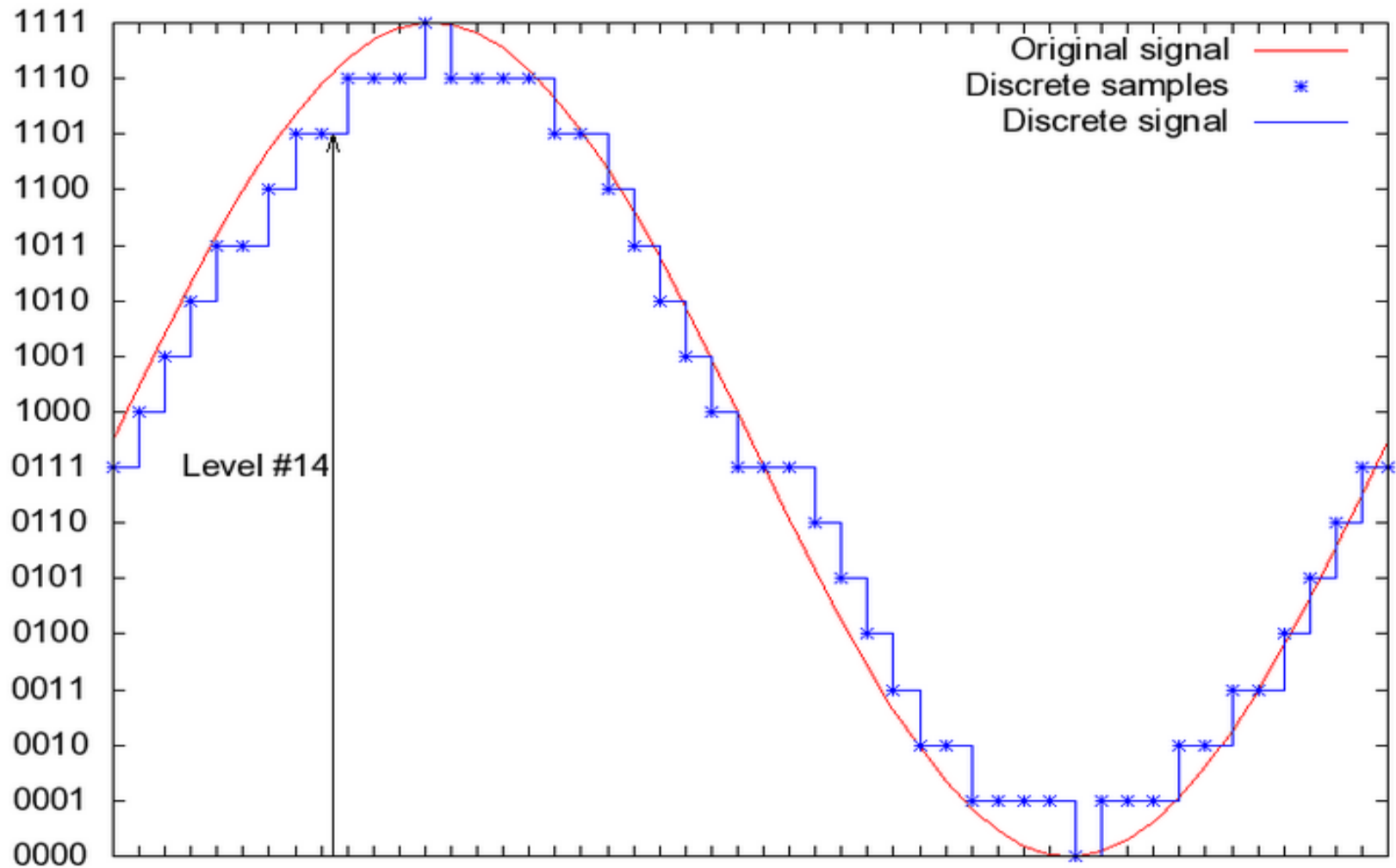
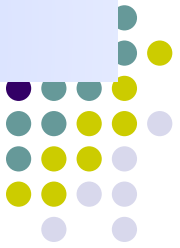


$$\begin{aligned} 1. \quad \text{SQR (dB)} &= 10 \log_{10} \left(\frac{A^2/2}{q^2/12} \right) \\ &= 7.78 + 20 \log_{10} \left(\frac{A}{q} \right) \end{aligned}$$

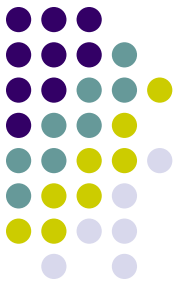
$$\begin{aligned} 2. \quad q &= (1) 10^{-(30-7.78)/20} \\ &= 0.078 \text{ V} \end{aligned}$$

3. Number of quantization Intervals is $1/0.078 = 13$ of each polarity yielding 26 intervals.
4. Number of bits $N = \log_2(26) = 4.7$ or approximately 5 bits per sample

SOLUTION /02



IDLE NOISE CHANNEL



- Noise occurring during **speech pauses is more objectionable than noise** occurring during speech.
- Idle Noise power is usually specified in absolute terms separate from quantization noise. Typical figure is **23dB_{rn}**.